

Velocity in SHM

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

$$v_{\max} = \omega A$$

Acceleration in SHM

$$a = \frac{dv}{dt} = -\omega^2 A \cos(\omega t + \phi)$$

$$a_{\max} = \omega^2 A$$

Energy in SHM

Kinetic Energy

$$K.E = \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t + \phi)$$

Potential Energy

$$P.E = \frac{1}{2} m \omega^2 A^2 \cos^2(\omega t + \phi)$$

Total energy, T.E. = K.E. + P.E.

$$T.E = \frac{1}{2} k A^2 [\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)]$$

Force law equation for SHM

$$F = -k(x), k = m\omega^2$$

$$\therefore \omega = \sqrt{\frac{k}{m}}$$

Displacement in SHM

$$x(t) = A(\cos \omega t + \phi)$$

A = Amplitude i.e., maximum displacement of particles.

Phase

Time varying quantity ($\omega t + \phi$)

Phase constant or Phase angle (ϕ)

- It depends upon velocity (v) and displacement of particle at $t = 0$

Simple Harmonic Motion is the simplest form of oscillatory motion.

SHM

Time period(T).

The smallest interval of time after which the motion is repeated

Frequency

The number of oscillations per second.

$$i.e., f = \frac{1}{T} = \frac{\omega}{2\pi}$$

Oscillations

Oscillations due to a spring

$$\text{Time period } T = 2\pi \sqrt{\frac{m}{k}}$$

Here, $k = \frac{F}{x}$ = spring constant

Simple pendulum

$$\text{Time period } T = 2\pi \sqrt{\frac{l}{mg}} = 2\pi \sqrt{\frac{l}{g}}$$

- Every oscillatory motion is periodic, but every periodic motion need not be oscillatory.
- To and from motion repeatedly about a fixed point in a definite interval of time.

Damped Oscillations

Oscillation of a body whose amplitude goes on decreasing with time.

Damping force $F_d = -bv$, b = damping constant

Displacement of damped oscillator

$$x = x_0 e^{-bt/2m} \sin(\omega t + \phi)$$

Amplitude decreases with time as

$$x = x_0 e^{-bt/2m}$$

Forced Oscillations

Driving force, $F(t) = F_0 \cos \omega_0 t$

Displacement, $x(t) = A \cos(\omega t + \phi)$

$$\text{Amplitude, } A = \frac{F_0}{\left[m^2 (\omega^2 - \omega_0^2)^2 + \omega^2 b^2 \right]^{1/2}}$$

$$\tan \phi = \frac{-v}{\omega_0 x_0}$$

Resonance

When frequency of external force is equal to natural frequency of oscillator.